The History of Branching Time

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How should time be represented graphically?

Circular time

Linear time

Branching time

Now
How should time be represented graphically?

Branching time

Branching time in literature:

Borges: *The Garden of Forking Paths*
Hans Christian Andersen: *The Story of a Mother*
….. probably many others ……

Hans Christian Andersen: *The Story of a Mother*

She looked into the well, and it made her glad to see how one life became a blessing to the world, for it was so kind and happy. Then she saw the other life, which held only sorrow, poverty, fear, and woe.

… "One life that you saw was your child's fate, your own child's future."
… And she bowed her head, as Death took her child to the unknown land.
Who invented the idea of branching time?

Henri Bergson (1859-1941)  A.N. Prior (1914-69)  Saul Kripke (born 1940)

…. an illustration of the process of deliberation… MO, a series of conscious states. At the state O two directions, OX and OY, equally open. ….
Henri Bergson: *Time and Free Will* (1910)

This is deceptive: Time as space

“We can analyse a thing but not a process; we can break up extensity, but not duration”
(p.219)
Who invented the idea of branching time?

“And I think it important that people who care for rigorism and formalism should not leave the basic flux and flow of things in the hands of existentialists and Bergsonians and others who love darkness rather than light, but we should enter this realm of life and time, not to destroy it, but to master it with our techniques.”

(Prior, undated note)

Arthur Norman Prior
(1914-69)

Who invented the idea of branching time?

Saul Kripke, Born Nov. 13, 1940.
Prof., Princeton, 1977-1998; distinguished professor of philosophy at The Graduate School and University Center of the City University of New York.
Basic axioms in modal logic + PC:
\[ L(p \supset q) \supset (Lp \supset Lq) \]
\[ Lp \supset p \]
\[ Mp \equiv \sim L\sim p \]
Rule: If \( p \) is provable then \( Lp \) is provable

Temporal interpretation of modal logic:
\( p \) is an arbitrary proposition the truth-value of which depends on a discrete and linear time.
\( Mp \) means: “\( p \) is or will be the case”
Prior uses “1” for true and “3” for false.

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S4-axioms added to PC:
\[ L(p \supset q) \supset (Lp \supset Lq) \]
\[ Lp \supset p \]
\[ Lp \supset LLp \]
Rule: If \( p \) is provable then \( Lp \) is provable

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\[
\begin{array}{cccccccccc}
 p & 1 & 3 & 1 & 3 & 1 & 3 & 3 & 1 & 1 & 3 & 3 & 3 \\
 Mp & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 3 & 3 & 3 & 3 \\
\end{array}
\]

Prior suggests that this use of \( M \) corresponds to S4.

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Saul Kripke’s letter to Prior: Sept. 3, 1958

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Prior suggests that this use of \( M \) corresponds to S4.

Saul Kripke rejects this. He shows that \( LMp \lor LM\neg p \) is valid in Prior’s system, but not provable in S4.
$p$ is an arbitrary proposition true of which depends on a discrete and linear time.

$Mp$ means: “$p$ is or will be the case”

Prior uses “1” for true and “3” for false.

Which temporal logic corresponds to S4?

Now in an indetermined system, we perhaps should not regard time as a linear series, as you have done. Given the present moment, there are several possibilities for what the next moment may be like -- and for each possible next moment, there are several possibilities for the next moment after that. Thus the situation takes the form, not of a linear sequence, but of a “tree”....
Saul Kripke’s letter to Prior: Sept. 3, 1958

$p$ is an arbitrary proposition true of which depends on a discrete and linear time. $Mp$ means: “$p$ is or will be the case” Prior uses “1” for true and “3” for false.

... a proposition is considered “necessary” if and only if it is and definitely always will be the case.

Saul Kripke’s letter to Prior Oct. 13, 1958

“Do you think a tensed logic is needed for scientific discourse? I should think that, for scientific discourse a tenseless logic may be preferable.”
Prior’s letter to Kripke Oct. 27, 1958

I do not see how indeterminism can be expressed in a tenseless language at all. For indeterminism asserts a certain difference between the future and the past …

Further development of branching time.
Kripke’s possible future corresponds to $MF(x)p$. - $P(x)p$ seems easy to represent. But how can $F(x)p, MF(x)p$ and $NF(x)p$ be understood in this framework?
Further development of branching time. Could there be alternatives pasts in a branching time system?

This would mean that it should be accepted that although the proposition $q$ was true yesterday, it may not be true today that it $q$ was the case yesterday. According to Prior this would not be acceptable.

The asymmetry between past and future

This belief of mine... is bound up with a belief in real freedom. One of the big differences between the past and the future is that once something has become past, it is, as it were, out of our reach - once a thing has happened, nothing we can do can make it not to have happened. But the future is to some extent, even though it is only to a very small extent, something we can make for ourselves.... if something is the work of a free agent, then it wasn’t going to be the case until that agent decided that it was.
[undated note]
Further development of branching time.
Kripke’s possible future corresponds to $MF(x)p$. - $P(x)p$ seems easy to represent. But how can $F(x)p$, $MF(x)p$ and $NF(x)p$ be understood in this framework?

Ockham’s (the medieval) view:

At least one chronicle is selected i.e. it represents what freely will be chosen. In this way $MF(x)p$, $F(x)p$ and $NF(x)p$ will be different.

Problem: How can we account for the difference between two different kinds of branches?

Further development of branching time.
Kripke’s possible future corresponds to $MF(x)p$. - $P(x)p$ seems easy to represent. But how can $F(x)p$, $MF(x)p$ and $NF(x)p$ be understood in this framework?

Prior’s Ockhamistic solution:

True-values have to be calculated relative to moments and chronicles: $Ock(p,t,c)$
In this way $MF(x)p$, $F(x)p$ and $NF(x)p$ will be different.
Further development of branching time. Kripke’s possible future corresponds to $MF(x)p$. $P(x)p$ seems easy to represent. But how can $F(x)p$, $MF(x)p$ and $NF(x)p$ be understood in this framework? The Peirce solution: No future or counterfactual branches are selected, but $F(x)p$ and is simply understood as $NF(x)p$. $P(x)q \lor P(x)\neg q$ holds. $F(x)q \lor F(x)\neg q$ does not hold.

Problem: How can we account for the fact that $F(x)p \lor F(x)\neg p$ is not a theorem is the Peirce system?

The two languages: Priors view

I believe that what we see as a progress of events is a progress of events, a coming to pass of one thing after another, and not just a timeless tapestry with everything stuck there for good and all... [Prior, Undated]
J.M.E. McTaggart (1866-1925): Two languages about time

Famous paper: “The Unreality of Time” [1908]

The A-language: past, present, future

The B-language: before, after, simultaneously with

The semantics (the B-language):
Time is a set of moments with an ordering relation, $<$, corresponding to before/after and

Any well formed proposition has a truth-value at a moment of time.

Definitions of the tenses:
\[
T(t, Fp) \equiv \exists t_1 : t < t_1 \land T(t_1, p)
\]
\[
T(t, Pp) \equiv \exists t_1 : t_1 < t \land T(t_1, p)
\]

Prior’s Ockhamistic system
\[
Ock(p, t, c) : \text{the truth-value of } p \text{ at } t \text{ for the chronicle } c \text{ through } t
\]
Two languages about time: The relations

Which of the two languages (if any) should be considered to be the basic (fundamental) language in temporal discourse?

Prior’s 4 grades:
1. The B-notions are more fundamental that the A-notions. Therefore, in principle the A-notions have to be defined in terms of the B-notions.
2. The B-notions are just as fundamental that the A-notions. The two sets of notions have to be treated on a par.
3. The A-notions are more fundamental that the B-notions. There is also a primitive and fundamental notion of (temporal) possibility. In principle the B-notions have to be defined in terms of the A-notions and the primitive notion of temporal possibility.
4. The A-notions are more fundamental that the B-notions. In principle the B-notions have to be defined in terms of the A-notions. Even the notion of temporal possibility can be defined on terms of the A-notions.
Prior’s idea of instant propositions:

Hybrid logic.

Instant propositions: \( a, b, c \) ....

(I1) \( \exists a: a \)
(I2) \( \neg \Box \neg a \)
(I3) \( \Box (a \supset p) \lor \Box (a \supset \neg p) \)

We can define \( T(a,p) \) as well as the before-after relation, \(<\):

(DT) \( T(a, p) \equiv_{def} \Box (a \supset p) \)
(DB) \( a < b \equiv_{def} \Box (a \supset Fb) \)

Prior’s view on time and logic:

Time is not an object, but whatever is real exists and acts in time... But this earlier-later calculus is only a convenient but indirect way of expressing truths that are not really about ‘events’ but about things ... [undated note]
Prior’s ideas of instant propositions and branching time:

An instant should be understood as a very rich proposition which implies everything which is true. In this way the whole branching time system is included in the instant. This means that any part of the system contains the whole system.

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The A-language and branching time

Prior made no use of Peirce’s Gamma Graphs. This is strange since it seems attractive from an A-theoretic point of view.

Challenge:
How can Prior’s tempo-modal systems be formulated in terms of Peirce’s Gamma Graphs?
… the question as to whether there are or could be unconnected time-series is a senseless one…. but these diagrams cannot represent time, as they cannot be translated into the basic non-figurative temporal language. [Prior 1967, p.199]