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# Deterministic versus indeterministic models: underdetermination and indirect evidence

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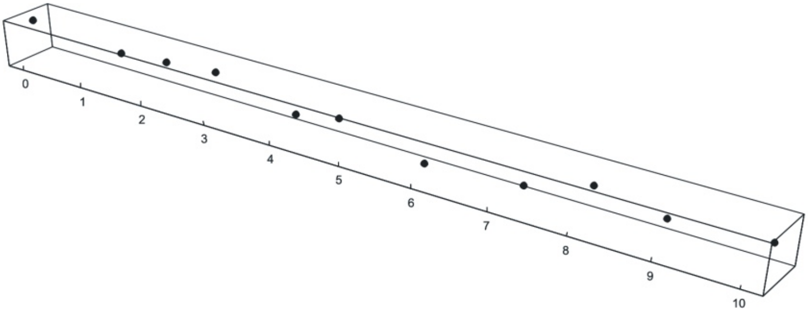
# Evolution of the position of a particle




Consider the physical process of the evolution of the position of a particle which bounces on mirrors. An observer obtains a sequence of observations.

# Central question

- Opinion might be that the observations only allow for a deterministic or an indeterministic model. However, recent results on observational equivalence show that this is not so.
- Is a deterministic model or an indeterministic model preferable relative to evidence?

- Deterministic and indeterministic models
- Observational equivalence
- Choice and underdetermination
- Which model is preferable? – a new argument
- Which model is preferable? – criticism of the literature
- Conclusion

# Deterministic models

- A model is deterministic iff any two solutions which agree at one time agree at all times.
- A deterministic model is a triple $(M, T_t, \mu)$. The set $M$ represents all possible states; $T_t(m)$ is the evolution of state $m$ after $t$ time units ($t$ integer-valued); $\mu$ is a probability measure
- A solution represents a possible evolution of the process over time, i.e., it is a sequence of states.

# Examples: Newtonian models of particles, billiards, gases

- Deterministic model of evolution of position of a particle which bounces on mirrors derived from Newtonian theory (baker's map).
- Other Newtonian models, such as of billiards, gases.

# Stochastic models

- A stochastic model $\{Z_t\}$ ($t$ integer-valued) with outcomes in $E$ represents a process that evolves according to probabilistic laws (indeterministic because probabilistic evolution).
- A realisation represents a possible evolution of the process over time, i.e., it is a sequence of outcomes.

# Example: stochastic model of throwing dice

- Stochastic model of throwing dice (each point of time dice are thrown and the outcomes are independent of each other) (Bernoulli process).

# Observations of deterministic and stochastic models

- A value is observed which is dependent on, but maybe different from, the actual state.
- Hence we can model an observation by an observation function, i.e., a function $\Phi : M \rightarrow M_O$ or $\Phi : E \rightarrow E_O$.
- In practice, observations are finite-valued (finite observational accuracy).

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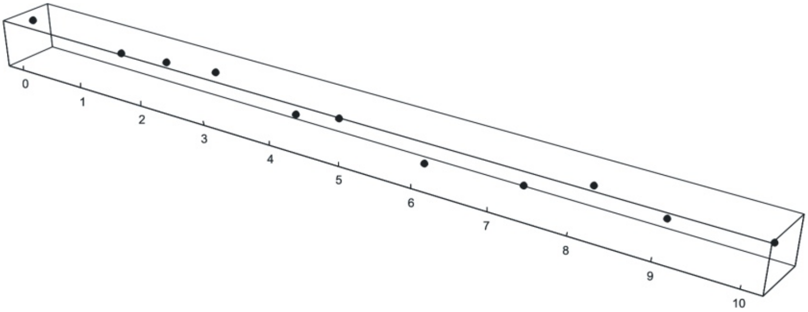
# Observational equivalence of deterministic and stochastic models

A deterministic model, when observed, and a stochastic model, when observed, are observationally equivalent iff:
(i) the possible observed states of the deterministic model and the possible observed outcomes of the stochastic model are the same;
(ii) the probability distributions over the observed solutions and the probability distributions over the observed realisations are the same.

# Results on observational equivalence I

- There are several results on observational equivalence.
- Werndl 2009, 2011: For many deterministic models including the Newtonian models of particles, billiards, gases: for any arbitrary finite observational accuracy there is observational equivalence to a nontrivial stochastic model.
- (More specifically, for any finite-valued observation function $\Phi$ the deterministic model $(M, T_t, \mu)$ is observationally equivalent to the nontrivial stochastic model $\{Z_t\}:=\{\Phi(T_t)\}$.)

# Evolution of particle and observational equivalence




For any observational accuracy the evolution of the position of the particle can be described by a deterministic and a stochastic model.

# Results on observational equivalence II

A question:

- Is it possible that deterministic models in science, when the respective systems are observed, are observationally equivalent to stochastic models in science (‘in science’ means derived or motivated with help of scientific theories)?

# Kolmogorov's conjecture

- Deterministic models which are observationally equivalent to stochastic models in science produce positive information. Deterministic models in science produce zero information.
- The Kolmogorov-Sinai entropy was introduced to measure the information produced by a system.
- Not true: several deterministic models in science (e.g., model of evolution of particle, models of billiards with convex obstacles) produce positive information.

# The answer is positive

- Several deterministic models in science (including model of evolution of particle, models of billiards with convex obstacles), when observed in certain ways (finite accuracy), are observationally equivalent to models of throwing dice.

# Results on observational equivalence II

Another question:

- Are there deterministic models in science which are observationally equivalent at any arbitrary observation level to stochastic models in science?

(Observational equivalence at every observation level means: for every finite-valued observation of a deterministic system, the probability distributions could have resulted from a stochastic process of a certain kind).

# Not with models of throwing dice

- Deterministic models in science are not observationally equivalent at every observation level to models of throwing dice.

# Markov model

- For a Markov model the next state only depends on the previous state of the process.

# The Answer is Positive

- Several deterministic models in science (including model of evolution of particle, models of billiard systems with convex obstacles) are observationally equivalent at every observation level to Markov models.
- Conclusion: there can be observational equivalence even if more and more restricted classes of deterministic and stochastic models are compared.

# Choice and underdetermination

- If there is observational equivalence, which model is preferable relative to all evidence?
- If none is preferable, there is underdetermination.
- (I concentrate on the case where the deterministic and the stochastic model describe the same level of reality.)

# Choice relative to in principle possible observations

- Relative to in principle possible observations (no limits, in principle, on observational accuracy) it is clear whether the deterministic or stochastic model is preferable.
- (If always finer observations can be made, the deterministic model is preferable; otherwise the stochastic model).
- So there is no underdetermination.

# Choice relative to currently possible observations

- In practice: choice relative to currently possible observations.
- Harder to find an answer because both models will agree with the currently possible observations.

# Indirect evidence

- Galileo's and Kepler's laws can be (approximately) derived from Newtonian theory. So predictions derived from Galileo's laws can provide indirect evidence for Kepler's laws.
- Laudan & Leplin (1991): indirect evidence can block underdetermination. E.g., suppose the same predictions are derivable from an hypothesis $H$ and from Kepler's law. Then, if there is indirect evidence only for Kelper's law, the evidence favours Kelper's laws over $H$.

## A new answer

- For Newtonian model of position of particle (and other deterministic Newtonian models, e.g., billiards, gases): There is more evidence for the deterministic model because it receives indirect evidence from Newtonian models which invoke similar physical assumptions.
- Consequently, the deterministic model is preferable, and there is no underdetermination.
- Generalisation: suppose that only one of the models is supported by indirect evidence. Then this model is preferable.

# Argument for the deterministic model

"It may well be true that there are some deterministic dynamical systems that, *when viewed properly*, display behavior indistinguishable from that of a genuinely stochastic process. For example, using the billiard table above, if one divides its surface into quadrants and looks at which quadrant the ball is in at 30-second intervals, the resulting sequence is no doubt highly random. But this does not mean that the same system, when viewed in a *different* way (perhaps at a higher degree of precision) does not cease to look random and instead betrays its deterministic nature [Hoefer 2008, original emphasis]".

# Criticism of Hoefer (2008)

- But for evolution of particle, billiards, gases: every finite-valued observation yields a nontrivial stochastic process (nontrivial transition probabilities).
- Even if there is an observation which yields trivial transition probabilities, the underlying dynamics might still be a (nontrivial) stochastic process.

# Another argument for the deterministic model

- The deterministic model is preferable because it is more informative.
- *"Some deterministic systems, when partitioned, generate stochastic processes. No one of these stochastic processes can, however, generate the deterministic flow. The deterministic flow is, if you like, a recipe for generating stochastic processes, none of which can, in return, generate its parent flow. [...] The deterministic model thus outstrips any single Markov model in its conceptual and predictive power." [Winnie 1998, 317].*

# Criticism of the information argument

- But: while it is true that the deterministic model provides more information, this information might not be of any value.
- Also: there are cases where a stochastic model is preferable because of indirect evidence (but deterministic model is more informative).
- Reasons based on 'providing more information' are non-evidential and thus not relevant here.

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# Conclusion

- There can be observational equivalence between deterministic and stochastic models.
- For deterministic model of position of particle and other Newtonian deterministic models: there is more evidence for the deterministic models because they receive indirect evidence from similar Newtonian models.
- Hoefer's argument and information argument that the deterministic model is preferable is unsatisfactory.

# Thank you!

Publications on this topic:


- Werndl, C. (2009). Are Deterministic Descriptions and Indeterministic Descriptions Observationally Equivalent?. *Studies in History and Philosophy of Modern Physics* 40, pp. 232–242.
- Werndl, C. (2011). On the Observational Equivalence of Continuous-Time Deterministic and Indeterministic Descriptions. *European Journal for the Philosophy of Science* 1 (2), pp. 193–225.
- Werndl, C. (2011). On Choosing Between Deterministic and Indeterministic Models: Underdetermination and Indirect Evidence. Forthcoming in: *Synthese*.