Logic Tea

Semantic and Syntactic Descriptions of Theories and Models

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Overview

- Two frameworks of theory analysis
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  - The received view
  - The semantic view
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- Two frameworks of theory analysis
  - The received view (a syntactic approach)
  - The semantic view (a semantic approach)
- Purported advantages of semantic approaches
  - Language independence
  - Connection to the world
  - Unintended models
  - Theory and Observation
  - Explicit axiomatization
  - Scientific models
Suppe’s look back

Twelve hundred persons were in the audience the night [the Received View] died. [...] The Received View had been under sustained attack for a decade and a critical mass of main protagonists had been assembled to fight it out. [...] Carl Hempel, a main developer of the Received View, was the opening speaker and was expected to present the Received View’s latest revision. Instead he told us why he was abandoning both the Received View and reliance on syntactic axiomatizations. [...] Suddenly we knew the war had been won[.]  
(Understanding Theories: An Assessment of Developments)
The warring factions

The received view

- as perceived by proponents (Carnap, Hempel, and others),
- as perceived by opponents.

The semantic view

- Suppes, Sneed, Stegmüller.
- Suppe, van Fraassen, French, da Costa, Ladyman.
- Giere, Hendry and Psillos.
Suppe’s summary of Carnap’s and Hempel’s view

- Theory given as a set of sentences.
- Sentences in first order logic.
- Bipartition of the vocabulary into observational terms and theoretical terms.
  - Observational terms are primitively interpreted.
  - Theoretical terms are only interpreted via the theory.
- Observations given in observational sentences.
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Two objects [... ] are called ‘isogeneous’ ['sphärenverwandt’] if there is a position in a propositional function for which the two object names are permissible arguments. For all other positions in any propositional function are then either both names permissible or both names impermissible arguments. This follows from the logical theory of types [...].

(Aufbau, § 29)

Let the structure of $L_T$ be such that it contains a typed logic with an infinite series of types $D^0, D^1, D^2$, etc.

(Beobachtungssprache und theoretische Sprache, 237)
The received view

Carnap on logic

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The semantic view

Relations between some semantic views

**Giere, Hendry and Psillos**: Non-linguistic.
- Scientific models instead of axiomatized theories.

**Suppes, Sneed, Stegmüller**: Set theoretic structures.
- Relation to model theory given by Przełęcki, Pearce, Tuomela.

**Suppe, van Fraassen, French, da Costa, Ladyman**: Model theoretic structures *without language.*
The semantic view

French and Ladyman on language independent structures

[If a model is] a map from the symbols of the syntax to elements of the structure[, then] the celebrated claim of the linguistic independence of considering models […] is simply not true.

[The] emphasis on structure is compatible with this definition of model theory from a contemporary textbook: “Model theory is the study of the construction and classification of structures within specified classes of structures.” (Reinflating the Semantic Approach, 114)

Part of Hodges’s definition of structure:

For each positive integer \( n \) [a structure contains] a set of \( n \)-ary relations on \( \text{dom}(A) \) (i.e. subsets of \( \text{dom}(A)^n \)), each of which is named by one or more \( n \)-ary relation symbols. […] (Model Theory, 2)
The semantic view

**What’s a structure?**

Da Costa, French, Ladyman, Bueno: \( \langle D, R_i \rangle_{i \in I} \).
- The index set gives a vocabulary.

Suppe and others: Tuples with domain as first element.
- A vocabulary can be associated with the ordering.
- Problematic for infinite sequences.

General problem: How to define model theoretic concepts without a vocabulary?
From no vocabulary to any vocabulary

Suggestion for a definition of a structure:

- The representative of a structure $\hat{A}$ is a mapping $f$ from a well-ordered index set to the image of an interpretation and a domain.
- Two mappings $f, f'$ represent the same structure iff their index sets are order isomorphic via $g$ and $f' = f \circ g$.

Definition of embedding:

- Use the position in the ordering instead of the names.

Upshot:

- Model $A$ can be embedded in $B$ iff for any ordering of their vocabularies, the resulting structure $\hat{A}$ can be embedded in $\hat{B}$.
- Structure $\hat{A}$ can be embedded in $\hat{B}$ iff they are represented by two models $A$ and $B$ with the same ordering of their vocabulary, and $A$ can be embedded in $B$. 
Syntactic and semantic approaches

Most prominent feature of

the received view: The description of theories in predicate logic (“the syntactic view”).

the semantic view: The description of theories by models.

To be compared in the following:

• Semantic approaches to theories
• Syntactic approaches: Descriptions by sets of first or higher order sentences.
Van Fraassen’s critique of the Received View

The syntactically defined relationships [between theory and observation] are simply the wrong ones. [...] It is hard not to conclude that those discussions of [...] ‘theoretical terms’, Craig’s theorem, [...] Ramsey and Carnap sentences, were one and all off the mark—solutions to purely self-generated problems, and philosophically irrelevant. The main lesson of twentieth-century philosophy of science may well be this: no concept which is essentially language-dependent has any philosophical importance at all. (Empirical Stance)
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Two readings:

1. Against the bipartition of the vocabulary into observational and theoretical terms.
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Two readings:

1. Against the bipartition of the vocabulary into observational and theoretical terms.

2. Against syntactic relations.
Suppe on semantic approaches

[Q]uantum theory can be formulated equivalently as wave mechanics or as matrix mechanics; whichever way it is formulated, it is the same theory, though its formulations as wave mechanics will constitute a collection of propositions which is different from the collection of propositions resulting from its formulations as matrix mechanics.

(Philosophical Understanding of Scientific Theories)

[U]nderstanding gained by focusing on models [...] challenges much conventional philosophical models[. . .] [B]y construing theories in terms of families of models, semantic analyses—and they alone—have real potential for parlaying such new philosophical wisdom into enhanced understanding of theories.  (Assessment of Developments)
Language dependence of semantic approaches

How far does the language independence go?

• Structural descriptions of theories allow introduction of a vocabulary:
  • Index sets of families.
  • Position in tuples.
  • Representatives of structures.

• Interesting theory changes change the structure.
  • Example: Hamiltonian mechanics introduces a Hamiltonian into Lagrangian mechanics.

How good is it to avoid names?

• $PV = kT \not\equiv TV = kP$
Language independence in syntactic approaches

Definitional equivalence:
$T$ is definitionally equivalent to $T'$ iff there are sets of explicit definition $D$ and $D'$ such that $T \cup D \models T' \cup D'$.

Advantages:

- Allows for a controlled change of names (‘Pressure’ $\mapsto$ ‘Druck’, ‘$P$’ $\mapsto$ ‘$V$’).

Definitional equivalence can be used in the semantic view as well.
Syntactic approaches

Interpretation of sets of sentences

- purely syntactically: Translation into primitively understood sentences.
- semantically: Identification of some intended interpretation in the metalanguage.

For intended interpretations to connect to the world, there have to be sets of worldly objects.
Semantic approaches

Use the metalanguage to describe structures with worldly objects (da Costa and French, Suppes?).

- Allows interpretation of a vocabulary.

Claim relevant isomorphy between non-worldly structures and worldly structures (Suppes?).

- Isomorphy to a worldly structure gives an interpretation of a vocabulary.

Identify structures that represent the world, leave relation to the world open (French and Ladyman).

- Syntactic translation into primitively understood sentences.
Suppe on unintended Models

A general problem [is] that the Löwenheim–Skolem theorem implies that [...] models must include both intended and wildly unintended models. Unintended models provide potential counterexamples.

Blocking them more concerns eliminating syntactical-approach artifacts than dealing with substantive analysis. [...] For example, Kitcher’s (1989) unification explanation account has a very simple idea. But he develops it syntactically spending most of the paper trying to block unintended consequences that are artifacts of his formalism. Most of his paper has little to do with developing his main idea. [...] (Understanding Theories: An Assessment of Developments)
Unintended and non-standard models

Ambiguity:

**Non-standard model**: Models that are not isomorphic to some intended model, but cannot be distinguished by first order logic.
- First order logic.
- Infinite domains.

**Unintended model**: A counterexample to a proposed explication of some intuitive concept.

Kitcher’s unintended models are not non-standard models, but inadequacies of the initial explication.
Non-standard models are a problem for a syntactic but not a semantic approach only when

- the syntactic description is in a first order language,
- the question cannot be phrased in (an extension of) the object language,
- the question is about infinitely many objects,
- intended interpretations do not distinguish between the standard and the non-standard model, and
- the syntactic and semantic descriptions are formally connected to worldly structures.
Observational objects and observational language

Two concepts of ‘observational’:

Received View: Bipartitioning of the vocabulary into observational and theoretical terms.
  - Also found in Sneed’s semantic approach.

Van Fraassen: Bipartitioning of the domain into observational and theoretical objects.
Suppes’s “Models of Data”

Models of the theory: Structures that fulfill the equations of the theory.

Models of the theory of experiment: Structures that fulfill the equations of the theory and (the description of?) the conditions of the experimental setup.

Models of the data: Structures that represent the experimental results and fulfill the equations of the theory with a sufficient goodness of fit (according to some statistical theory).

In syntactic terms:
Models of the theory: Models of the syntactic theory.
Models of the data: Models of the description of the experimental results that are also models a description of a good fit, derived from the theory itself and a statistical theory.
Suppes and Stegmüller on explicit axiomatization

Suppose we want to give a standard formalization of elementary probability theory. [...] Finally, after stating a group axioms on sets, and another group on the real numbers, we are in a position to state the axioms that belong just to probability theory as it is usually conceived. In this welter of Axioms, those special to probability can easily be lost sight of.

More important, is senseless and uninteresting continually to repeat these general axioms on sets and on numbers whenever we consider formalizing a scientific theory. No one does it, and for good reason.

(Suppes: Axiomatic Methods in Science, 207f)

To carry out the programme suggested by statement view is not humanly possible.

(Stegmüller: The Structuralist View, 5)
Carnap on deduction

It would, of course, be practically impossible to give each deduction which occurs the form of a complete derivation in the logical calculus [. . . .] But it is essential that this dissolution is theoretically possible and practically possible for any small part of the process. [. . . .] [In a controversy, it] will be sufficient to expand the the critical part of the controversial deduction to the degree required by the situation. (Foundations of Logic and Mathematics, 179)

This can just as well be applied to axiomatizations.
Suppes on models

[M]any physicists want to think of a model of a the orbital theory of the atom as being more than a certain kind of set-theoretical entity. [...] [T]here is no real incompatibility in these two viewpoints [...] for the physical model may be simply taken to define the set of objects in the set-theoretical model. [...] Ultimately it is the mathematical theory of Maxwell which has proved important, not the physical image of an ether behaving like an elastic solid.

(Meaning and Uses of Models, 290ff)

[On scientific models:] In the language of logicians, it would be more appropriate to say that rather than constructing a model [scientists] are interested in constructing a quantitative theory to match the intuitive ideas of the original theory.

(293)
Hempel on models

[Alternative interpretations of theories] may make for “intellectual economy” [and] facilitate one’s grasp of a set of explanatory laws or theoretical principles [. . . .] [Well-chosen analogies or models may prove useful “in the context of discovery,” i. e., they may provide effective heuristic guidance [because a model] may suggest extensions of the analogy on which it was originally based. Aspects of Scientific Explanation (440f)

[On scientific models:] [A] theoretical model of this kind has the character of a theory with a more or less limited scope of application. (446)
Hempel on syntactic axiomatizations

“[Hempel] told us why he was abandoning both the Received View and reliance on syntactic axiomatizations.”

This extensive theoretical use of antecedent terms appears to me to throw into question the conception of the internal principles of a theory as an axiomatized system whose postulates provide “implicit” definitions for its extralogical terms. […] Hence, the theoretical “calculus” of a theory of this kind cannot be regarded as strictly formalized, uninterpreted system, and the concepts of model theory cannot be applied to it without qualifications.

(Formulation and Formalization of Scientific Theories, 251)
Conclusion

Vindicating syntactic approaches

**Language independence:** Independence from vocabulary is not straightforward in semantic approaches, and can be had in a more controlled and more general way by syntactic approaches.

**Connection to the world:** Semantic solutions work for syntactic approaches and vice versa.

**Unintended models:** The technical problem is not trouble for explications, and only occurs in very specific cases.

**Observation:** Commonly suggested connections between theory and observation work for both approaches.

**Explicit axiomatization:** Balance between explicitness and usability in both approaches.

**Scientific models:** The same status in either approach.
Structure

Definition
A representative of a structure $\hat{\mathcal{A}}$ is a pair $\langle a, \prec \rangle$, where $a : A \rightarrow \mathcal{A}(\{\forall\} \cup \forall')$ is a mapping from the index set $A$ to the image of an interpretation $\mathcal{A}$, and $\prec$ is a well-ordering of $A$ such that the smallest element of $A$ is mapped to $\mathcal{A}(\forall)$ by $a$. Two pairs $\langle a, \prec \rangle$ and $\langle b, \prec' \rangle$ represent the same structure, $[\langle a, \prec \rangle] = [\langle b, \prec' \rangle]$ iff there is an order isomorphism $f : A \rightarrow B$ and $a \circ f = b$. 
Embedding for structures

A structure $\hat{A}$ with domain $D$ can be embedded in a structure $\hat{B}$ with domain $E$ if and only if for any two representatives $[\langle a, \prec \rangle] = \hat{A}$ and $[\langle b, \prec' \rangle] = \hat{B}$, there is an order isomorphism $g : A \rightarrow B$ from $a$’s index set to $b$’s index set and a one-to-one mapping $h : D \rightarrow E$ such that

1. for all $c \in A$ mapped to constants by $a$, $h(a(c)) = b(g(c))$,

2. for all $f \in A$ mapped to $n$-ary functions by $a$ and all $x_1, \ldots, x_n \in D$, $h\left( a(f)(x_1, \ldots, x_n) \right) = b\left( g(f) \right)(h(x_1), \ldots, h(x_n))$, and

3. for all $R \in A$ mapped to $n$-ary relations by $a$ and all $x_1, \ldots, x_n \in D$,
   
   $\langle x_1, \ldots, x_n \rangle \in a(R) \iff \langle h(x_1), \ldots, h(x_n) \rangle \in b(g(R))$,

$h$ is called an embedding of $\hat{A}$ in $\hat{B}$. If $h$ is onto, $\hat{A}$ and $\hat{B}$ are called isomorphic.
Embedding between representatives of structures

\[ C \xrightarrow{i} A \xrightarrow{a} \mathcal{A} (\{\forall\} \cup \mathcal{V}) \]
\[ B \xrightarrow{j} \mathcal{B} (\{\forall\} \cup \mathcal{V}') \]
\[ k = j \circ g \circ i \]
\[ D \xrightarrow{d = b \circ j^{-1}} E = \mathcal{B}(\forall) \]
\[ D = \mathcal{A}(\forall) \]
\[ g \]
\[ h \]
Embeddings for structures and for models

Structure $\hat{\mathcal{A}}$ can be embedded in $\hat{\mathcal{B}}$ iff there are interpretations $\mathcal{A}$ and $\mathcal{B}$ and a well-ordering $\prec$ such that $\hat{\mathcal{A}} = [\langle \mathcal{A}, \prec \rangle]$, $\hat{\mathcal{B}} = [\langle \mathcal{B}, \prec \rangle]$, and $\mathcal{A}$ can be embedded in $\mathcal{B}$.

Interpretation $\mathcal{A}$ can be embedded in interpretation $\mathcal{B}$ iff $[\langle \mathcal{A}, \prec \rangle]$ can be embedded in $[\langle \mathcal{B}, \prec \rangle]$ for some well-ordering $\prec$. 